

$$D: (-\infty, \infty) \quad R: (0, \infty)$$

$$D: (0, \infty) \quad R: (-\infty, \infty)$$

\*Inverse of each other

$$\log_e x = \ln x$$

$$\log_b A = e \leftrightarrow b^e = A$$

$$\ln A = x \leftrightarrow e^x = A$$

b is base, e is exponent, A is answer

- 1) The domain of the logarithmic function  $f(x) = \log_a x$  is  $\{x | x > 0\}$  or  $(0, \infty)$ .
- 2) The graph of the  $f(x) = \log_a x$ ,  $a > 0$ ,  $a \neq 1$ , has an x-intercept equal to 1 and no y-intercept.

Choose the correct answer below.

☒ True

- 3) Determine the domain of  $f(x) = \log_2(x + 1)$ .

Choose the correct answer below.

☐  $(-\infty, \infty)$

☒  $(-1, \infty)$

- 4) Change the exponential statement to an equivalent statement involving a logarithm.

$$8 = 2^3 \quad b^e = A \quad b = 2 \quad e = 3 \quad A = 8 \quad \log_b A = e$$

The equivalent logarithmic statement is  $\log_2 8 = 3$ . (Type an equation.)

- 5) Change the exponential statement to an equivalent statement involving a logarithm.

$$4.5 = a^5$$

The equivalent logarithmic statement is  $\log_a 4.5 = 5$ .

- 6) Change the exponential statement to an equivalent statement involving a logarithm.

$$9.7 = 7^x$$

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Evaluate the expression without using a calculator.  $\log_7 9.7 = x$ .

$$\log_3 1$$

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$$\log_3 1 = 0$$

- 7) Change the exponential statement to an equivalent statement involving a logarithm.

$$e^x = 11$$

$$\log_e 11 = x \text{ where } \log_e = \ln$$

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The equivalent logarithmic statement is  $\ln 11 = x$ . (Type an equation.)

- 8) Change the logarithmic statement to an equivalent statement involving an exponent.

$$\log_9 81 = 2$$

$$\text{Log}_b A = e \quad b^e = A$$

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The equivalent exponential statement is  $9^2 = 81$ . (Type an equation.)

- 9) Change the logarithmic statement to an equivalent statement involving an exponent.

$$\log_a 5 = 6$$

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The equivalent exponential statement is  $a^6 = 5$ . (Type an equation.)

- 10) Change the logarithmic statement to an equivalent statement involving an exponent.

$$\log_8 4096 = x$$

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The equivalent exponential statement is  $8^x = 4096$ . (Type an equation.)

- 11) Change the logarithmic statement to an equivalent statement involving an exponent.

$$\ln 11 = x$$

---

The equivalent exponential statement is  $e^x = 11$ . (Type an equation.)

$$b^0 = 1 \quad x^{-1} = \frac{1}{x} \quad x^{1/2} = \sqrt{x}$$

- 12) Evaluate the expression without using a calculator.

$$\log_3 1 \quad 3^x = 1 \quad x = 0$$


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$$\log_3 1 = 0$$

- 13) Find the exact value of the logarithm without using a calculator.

$$\log_5 25 \quad 5^x = 25 \quad x = 2$$


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$$\log_5 25 = 2$$

- 14) Find the exact value of the logarithm without using a calculator.

$$\log_{1/2} 16 \quad \frac{1^x}{2} = 16 \quad 2^4 = 16$$


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$$\log_{1/2} 16 = -4 \text{ (Type an integer or a simplified fraction.)}$$

\*fraction to a whole number is negative exponent

- 15) Find the exact value of the logarithm without using a calculator.

$$\log_8 \sqrt{8} = x \quad 8^x = \sqrt{8} \quad x^{1/2} = \sqrt{x}$$

Same bases  $\rightarrow 8^x = 8^{1/2}$  then  $x = \frac{1}{2}$

- 16) Find the exact value of the logarithm without using a calculator.

$$\log_{\sqrt{7}} 49 \quad \sqrt{7}^x = 49$$


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$$\log_{\sqrt{7}} 49 = 4$$

Same bases  $\rightarrow 7^{1/2x} = 7^2$  then

$$\frac{1}{2}x = 2 \quad x = 4$$

- 17) Find the exact value of the logarithm without using a calculator.

$$\ln \sqrt[4]{e} \quad \text{*} \text{Ine cancel each other out and the answer is the exponent}$$


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$$\ln \sqrt[4]{e} = \frac{1}{4} \text{ (Type an integer or a simplified fraction.)}$$

Square root is a fractional exponent

- 18) Find the domain of the function. only use horizontal shift, which is in **parenthesis**

$$h(x) = \ln(x - 9) \quad \text{right 9}$$

The domain of h is  $(9, \infty)$ .

(Type your answer in interval notation.)

- 19) Domain of  $9 - 2\log_3\left[\frac{x}{6} - 5\right]$  only use horizontal shift, which is in **brackets**
- $$\frac{x}{6} - 5 > 0 \quad x > 30 \quad (30, \infty)$$

- 20) Graph the following function and its inverse on the same Cartesian plane.

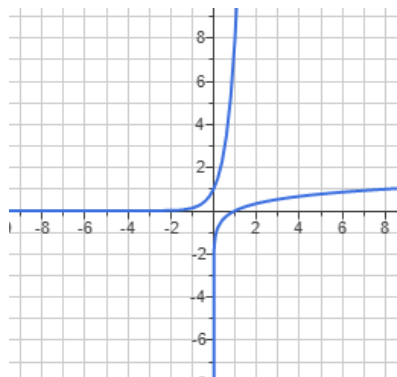
$$f(x) = 8^x; f^{-1}(x) = \log_8 x$$

Use the graphing tool to graph the function.

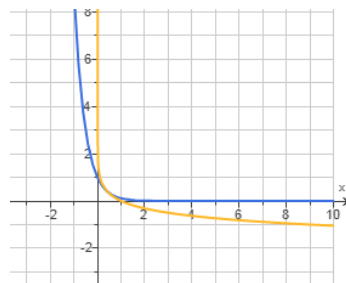


Vertical Stretch or Compression (a)	Horizontal Stretch or Compression (a)
1	1
Vertical Shift (k)	Horizontal Shift (h)
0	0
<input type="checkbox"/> Reflect over x-axis	<input type="checkbox"/> Reflect over y-axis
Base	
8	

Click and change base to 8 then and change base to 8  
MAKE TWO GRAPHS

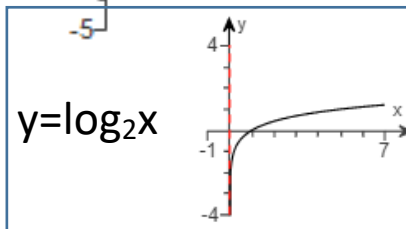
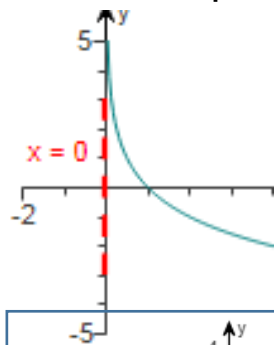


- 21)  $f(x) = \left(\frac{1}{7}\right)^x$   $f^{-1}(x) = \log_{\frac{1}{7}} x$   
Inverse is yellow



Choose both shapes and make base  $\frac{1}{7}$  for both graphs

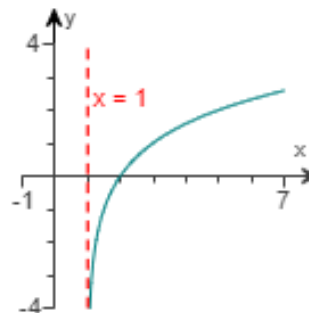
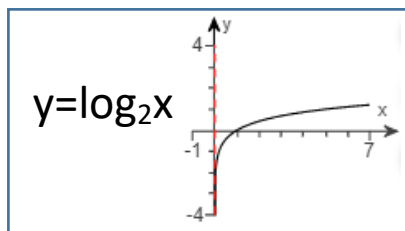
22) Find the equation:



- A.  $y = -\log_2(-x)$
- B.  $y = \log_2(-x)$
- C.  $y = \log_2 x - 1$
- D.  $y = -\log_2 x$
- E.  $y = \log_2 x$
- F.  $y = 1 - \log_2 x$
- G.  $y = \log_2(1 - x)$
- H.  $y = \log_2(x - 1)$

*Flips across the x axis*

23) The graph of a logarithmic function is given. Match the graph to its function.



*Shifts right 1*

Which function matches the graph?

- ☐ A.  $y = -\log_2 x$
- ☐ B.  $y = 1 - \log_2 x$
- ☐ C.  $y = -\log_2(-x)$
- ☐ D.  $y = \log_2 x$
- ☒ E.  $y = \log_2(-x)$
- ☒ F.  $y = \log_2(x - 1)$

EXAMPLE:  $\ln(x+4)-3$       inverse:  $e^{x+3} - 4$

EXAMPLE:  $\log_7(x-2) + 5$       inverse:  $7^{x-5} + 2$

Horizontal and vertical shifts switch and change the sign  
\*Domain and Range switch.

24) Use the given function  $f$  to answer parts (a) th

$$f(x) = \ln(x-3)$$

Right 3 converts to up 3

(a) Find the domain of  $f$ .

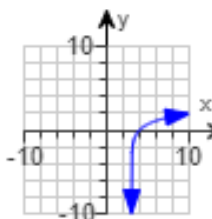
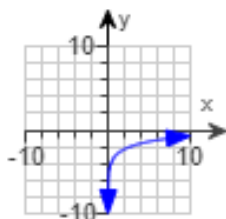
The domain of  $f$  is  $(3, \infty)$ .

(Type your answer in interval notation.)

(b) Graph  $f$ . Choose the correct graph below.

Right 3

☒ B.



(c) From the graph, determine the range and an

The range of  $f$  is  $(-\infty, \infty)$ .

(Type your answer in interval notation.)

Determine the vertical asymptote of  $f$ . Select the answer box to complete your choice.

☒ A. The vertical asymptote of  $f$  is  $x = 3$ .

(d) Find  $f^{-1}$ , the inverse of  $f$ .

$$f^{-1}(x) = e^x + 3 \quad (\text{Simplify your answer.})$$

(e) Find the domain and range of  $f^{-1}$ .

The domain of  $f^{-1}$  is  $(-\infty, \infty)$ .

(Type your answer in interval notation.)

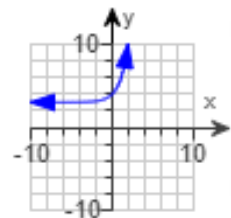
The range of  $f^{-1}$  is  $(3, \infty)$ .

(Type your answer in interval notation.)

Up 3

(f) Graph  $f^{-1}$ . Choos

☒ A.



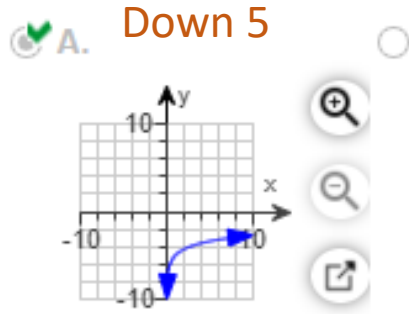
25) Use the given function  $f$  to an

$$f(x) = -5 + \ln x$$

(a) Find the domain of  $f$ .

The domain of  $f$  is  $(0, \infty)$ .  
(Type your answer in interval

(b) Graph  $f$ . Choose the corre



(c) From the graph, determine the range and

The range of  $f$  is  $(-\infty, \infty)$ .  
(Type your answer in interval notation.)

Determine the vertical asymptote of  $f$ . Select answer box to complete your choice.

☒ A. The vertical asymptote of  $f$  is  $x = 0$ .

down 5 converts to right 5

(d) Find  $f^{-1}$ , the inverse of  $f$ .

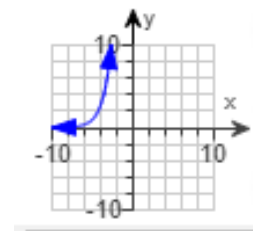
$$f^{-1}(x) = e^{x+5} \text{ (Simplify your answer.)}$$

(e) Find the domain and range of  $f^{-1}$ .

The domain of  $f^{-1}$  is  $(-\infty, \infty)$ .  
(Type your answer in interval notation.)

The range of  $f^{-1}$  is  $(0, \infty)$ .  
(Type your answer in interval notation.)

(f) Graph  $f^{-1}$ . Choose the correct graph below.  
Left 5



26)  $f(x) = -2 + \log_3(x + 4)$

Domain  $(-4, \infty)$  Range  $(-\infty, \infty)$

Find  $f^{-1}(x)$  algebraic way:

$$x = -2 + \log_3(y + 4)$$

$$x + 2 = \log_3(y + 4)$$

$$3^{x+2} = y + 4$$

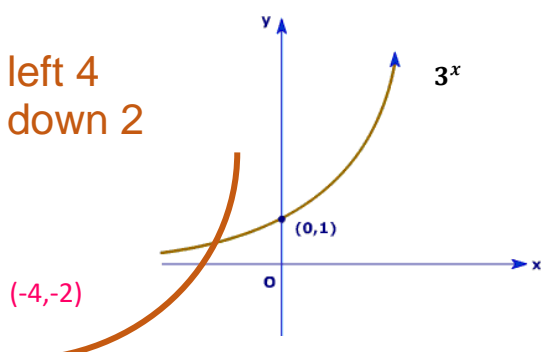
$$3^{x+2} - 4 = f^{-1}(x) \quad \text{Domain } (-\infty, \infty)$$

$$\text{Range } (-4, \infty)$$

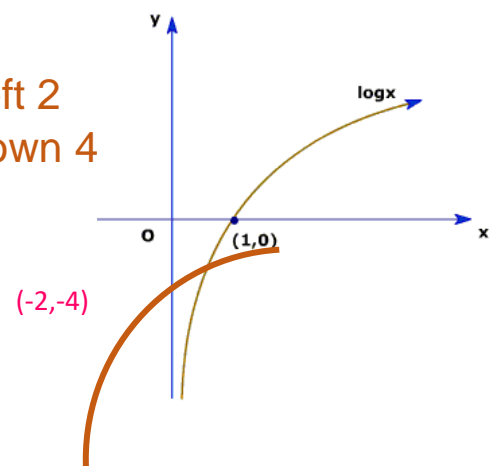
left 4 down 2 switch position and sign

**SHORTCUT: right 2 down 4**  
$$f^{-1}(x) = 3^{x+2} - 4$$

left 4  
down 2



left 2  
down 4



coordinates switch

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

27) Solve:  $\log_6 x = 3$

Another way:  $6^{\log_6 x} = 6^3$

change to exponential form:  $6^3 = x$

$$x = 216$$

28)  $f(x) = \log_3(9x - 2) = 4$

Another way:  $3^{\log_3(9x-2)} = 3^4$

change to exponential form:  $3^4 = 9x - 2$

$$9x - 2 = 81$$

$$x = \frac{83}{9}$$

29) Solve the equation.

$$\log_x 64 = 2$$

$$x^2 = 64$$

The solution set is  $\{8\}$ .

30) Solve the equation.

$$\log_6 216 = x$$

$$6^x = 216$$

The solution set is  $\{3\}$ .

31)  $f(x) = \log_5 125 = 3x + 1$

$$5^3 = 125$$

$$3 = 3x + 1$$

$$2 = 3x$$

$$x = \frac{2}{3}$$

32)  $f(x) = e^{2x} = 3$  take ln of both sides **line = 1**

$$\ln e^{2x} = \ln 3$$

$$2x = \ln 3$$

$$x = \frac{\ln 3}{2}$$



33)  $f(x) = e^{8x-3} = 2$  take ln of both sides **lne = 1**

$$\ln e^{8x-3} = \ln 2$$

$$8x - 3 = \ln 2$$

$$8x = \ln 2 + 3 \quad x = \frac{\ln 2 + 3}{8}$$

34)  $f(x) = \log_4(x^2 - 6) = 3$  change to exponential form

$$x^2 - 6 = 4^3 \quad x^2 = 70 \quad x = -\sqrt{70}, \sqrt{70}$$

35)  $f(x) = \log_9 729^x = -3$  change to exponential form

$$729^x = 9^{-3}$$

$$9^{3x} = 9^{-3} \quad x = -1$$

36)  $f(x) = 5 \cdot 10^{4-x} = 12$  divide by 5 first

$$10^{4-x} = \frac{12}{5} \quad \text{take } \log_{10} \text{ on both sides}$$

$$\log_{10} 10^{4-x} = \log_{10} \left( \frac{12}{5} \right)$$

$$4 - x = \log_{10} \left( \frac{12}{5} \right) \quad \text{move 4 to the right}$$

$$-x = \log_{10} \left( \frac{12}{5} \right) - 4 \quad \text{divide by -1}$$

$$x = -\log_{10} \left( \frac{12}{5} \right) + 4$$

37) Suppose that  $G(x) = \log_2(2x + 2) - 3$   $2x+2 = 0$   $2x = -2$

(a) What is the domain of G?  $(-1, \infty)$

(b) What is G(1)? What point is on the graph? Plug in 1 for x

$$\log_2(2(1) + 2) - 3$$

$$\log_2(4) - 3$$

$$2^2 = 4$$

$$2 - 3 = -1 \quad \text{where point is } (1, -1)$$

(c) If  $G(x) = 2$ , what is x? What is the point?

$$\log_2(2x + 2) - 3 \quad \text{set equal to 2}$$

$$\log_2(2x + 2) - 3 = 2 \quad \text{move -5 to the right}$$

$$\log_2(2x + 2) = 5 \quad 4^0 = 1$$

$$2x+2=2^5 \quad 2x=30 \quad x=15 \quad \text{where point is } (15, 2)$$

(d) Give the zero of  $G(x)$  *\*set it equal to zero*

$$\log_2(2x + 2) - 3 = 0$$

$$\log_2(2x + 2) = 3$$

$$2x+2=2^3 \quad 2x=6 \quad x = 3$$

38) The pH of a chemical solution is given by the formula

$\text{pH} = -\log_{10}[\text{H}^+]$  where  $[\text{H}^+]$  is the concentration of hydrogen ions in moles per liter. Values of pH range from 0 (acidic) to 14 (alkaline)

a) What is the pH of a solution for which  $[\text{H}^+]$  is 0.1? **1**

b) What is the pH of a solution for which  $[\text{H}^+]$  is 0.01? **2**

c) What is the pH of a solution for which  $[\text{H}^+]$  is 0.001? **3**

d) What happens to the pH as hydrogen decreases? **pH increases**

e) Determine the hydrogen ion concentration of a solution with  $\text{pH} = 3.9$

$$3.9 = -\log_{10}[\text{H}^+]$$

$$-3.9 = \log_{10}[\text{H}^+]$$

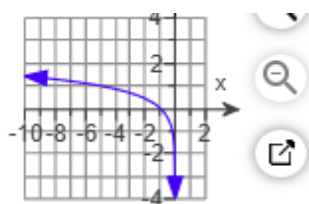
$$10^{-3.9} = [\text{H}^+] \text{ .000126}$$

e) Determine the hydrogen ion concentration of a solution with  $\text{pH} = 7.8$

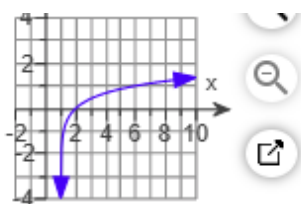
$$7.8 = -\log_{10}[\text{H}^+]$$

$$-7.8 = \log_{10}[\text{H}^+]$$

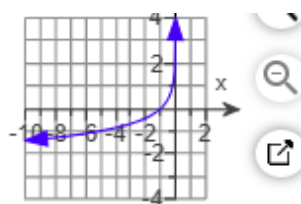
$$10^{-7.8} = [\text{H}^+] \text{ } 1.6 \times 10^{-8}$$



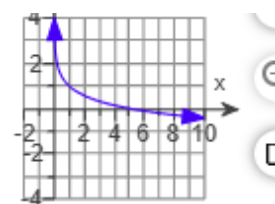
$$y = \log_5(-x)$$



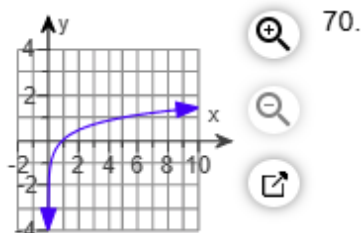
$$y = \log_5(x - 1)$$



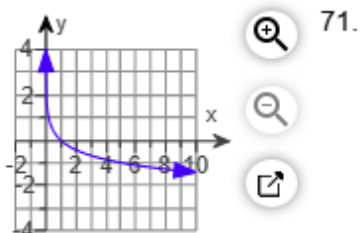
$$y = -\log_5(-x)$$



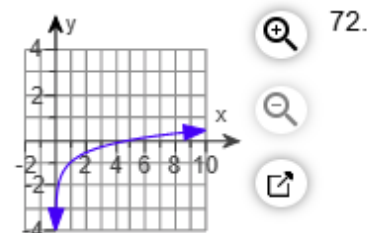
$$y = 1 - \log_5 x$$



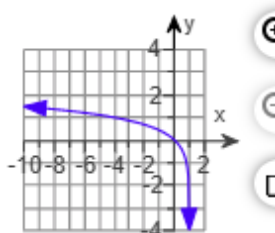
$$y = \log_5 x$$



$$y = -\log_5 x$$



$$y = \log_5 x - 1$$



$$y = \log_5(1 - x)$$